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作品名稱 **Locus of the Points on Circumference of the n-th Circle that Formed by Moving the Center of any Radius Circles on the Outermost Circumference of Preceding set of Circles**

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關鍵詞 **Locus**

作者照片


Circumference of Preceding set of Circles
Methods
Results and Discussion
Conclusion
Set of 3 circles

Research question

According to the study, Engare is a puzzle game about motion and geometry which players will have to solve problems by using geometric shapes to create images. We found a different level in the Engare game. There is a pattern of three consecutive sticks with pivot point at the end of the preceding stick which the ends of each bar can be rotated around the pivot point. In this stage, players have to adjust the angular velocity or length of each stick to achieve the game's visualization. We are interested in finding the locus equation by studying the equation from the epicycloid and the angular velocity to find the relationship of the ratio of velocity to the locus.

Purposes

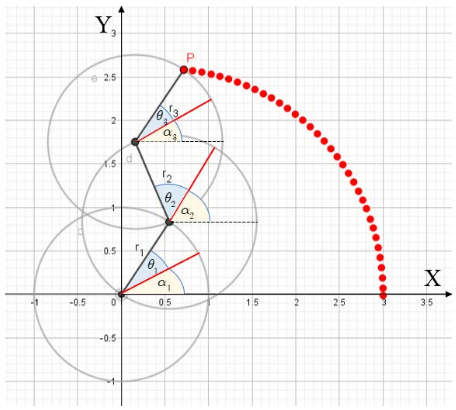
To find locus equation of the point on circumference of the n -th circle that formed by moving the center of any radius circles on the outermost circumference of preceding set of circles.

Hypothesis

locus equation of the point on circumference of the n -th circle that formed by moving the center of any radius circles on the outermost circumference of preceding set of circles is in polar coordinates using angular velocity.

Definitions

1.The locus of a points on the outermost circumference: The set of points on the tangent line of the curve on the plane which the distance from the three points are aligned to the same loop again.



2. Point $P(x, y)$ **:** The point where the locus is built on the outermost circumference.

3. Set of Circles (ic) **:** The center of the first circle is at the origin and the center of the next circle is on the circumference of the preceding circle.

4. Angular velocity of point (ω) **:** The distance of moving the point relates to the time of the center moving along the circumference of the preceding circle.

5. Ratio of angular velocity $\omega_1 : \omega_2 : \omega_3$ **:** Angular velocity ratio of point is an innermost point is ω_1 middle point is ω_2 outermost is ω_3 .

6. Starting angle (α_i) **:** A starting angle between the radius of i -th circle and X-axis.

Figure 1 locus from set of 3 circles $(3c)$ and α_1, α_2, $\alpha_3 > 0$

Scope of the project study

1. Study the locus of a point that rotates around the circumference of the outermost circle in the same direction of rotation.
2. Study the ration of locus that occurred from the ratio of the angular velocity to $\omega_1 : \omega_2 : \omega_3$ when $1 \leq \omega_1 \leq 10$, $1 \leq \omega_2 \leq 10$ and $1 \leq \omega_3 \leq 10$

Methodology

There are 3 steps involved in this project

Step 1 Create the motion of a fixed point in the GSP.

1.1 Create a set of circles with 2 and 3 circles.

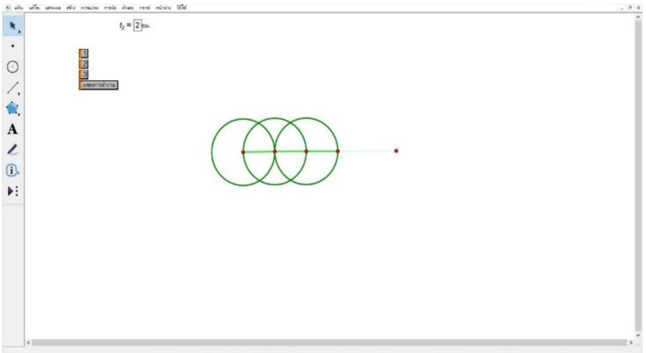


Figure 2 create a set of circles and set the angular velocity.

1.2 Set the angular velocity of each point sequentially such as the ratios of the angular velocity are $\omega_1 : \omega_2 : \omega_3$ when $1 \leq \omega_1 \leq 10$, $1 \leq \omega_2 \leq 10$ and $1 \leq \omega_3 \leq 10$

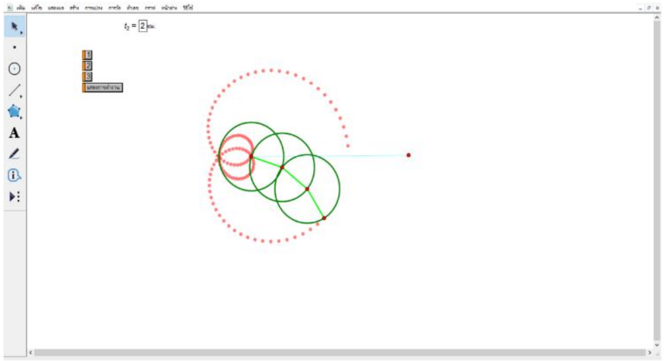


Figure 3 create locus

Step 2 Analyze the motion of a fixed point.

2.1 Storage traces of each aspect ratios in an excel program.

2.2 Observe the same motion.

Out of a total of 1,000 locus, divided into 172 groups with different locus characteristics. Each group had the same locus characteristics, but different ratio of angular velocity

2.2.1 The locus formed by the motion of a point with a ratio of angular velocity 1: 1: 1, 2: 2: 2, 3: 3: 3, …, 10: 10: 10 will be circular therefore, all ratio of angular velocity $\omega : \omega : \omega$ where $\omega \in I^+$ will be circular

2.2.2 The locus formed by the motion of a point with a ratio of angular velocity 1: 1: 2, 1: 2: 1, 2: 1: 1 is the same pattern and 1: 2: 3, 1: 3: 2, 2: 1: 3, 2: 3: 1, 3: 1: 2, 3: 2: 1 is the same pattern therefore, all the permutations of $\omega_1, \omega_2, \omega_3$ ratio of angular velocity $\omega_1 : \omega_2 : \omega_3$ is the same pattern

2.2.3 The locus formed by the motion of a point with a ratio of angular velocity 1: 1: 2, 2: 2: 4, 3: 3: 6, 4: 4: 8, 5: 5: 10 is the same pattern and locus formed by the motion of a point with a ratio of angular velocity 1: 2: 3, 2: 4: 6, 3: 6: 9 is the same pattern therefore, $\omega_1 : \omega_2 : \omega_3$ is minimum ratio, then the locus from the motion traces of the angular velocity ratio $k\omega_1 : k\omega_2 : k\omega_3$ is the same pattern where $k \in I^+$

Figure 5 Example of locus characteristics of a set of 3 circles

Figure 4 Example of locus characteristics of a set of 3 circles with different pattern of 172 groups

Step 3 Create the locus equation

3.1 Create locus equation of the points on circumference of the 2-nd circle that formed by moving the center of any radius circles on the outermost circumference of preceding set of circles.

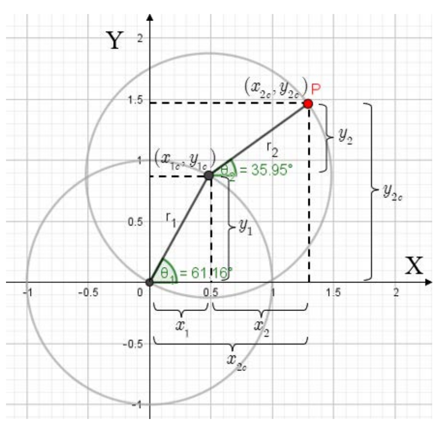


Figure 6 set of 2 circles

3.2 Create locus equation of the points on circumference of the 3-rd circle that formed by moving the center of any radius circles on the outermost circumference of preceding set of circles.

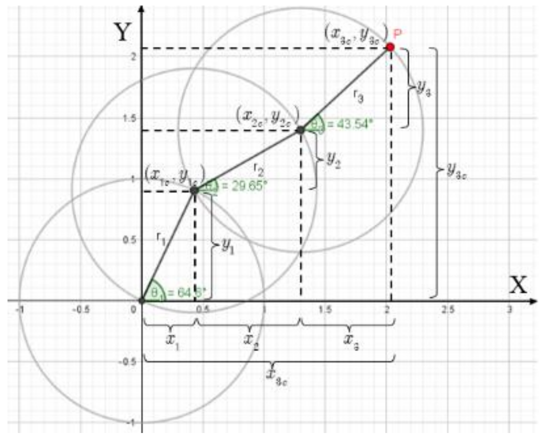


Figure 7 set of 3 circles

3.3 Create locus equation of the points on circumference of the n -th circle that formed by moving the center of any radius circles on the outermost circumference of preceding set of circles.

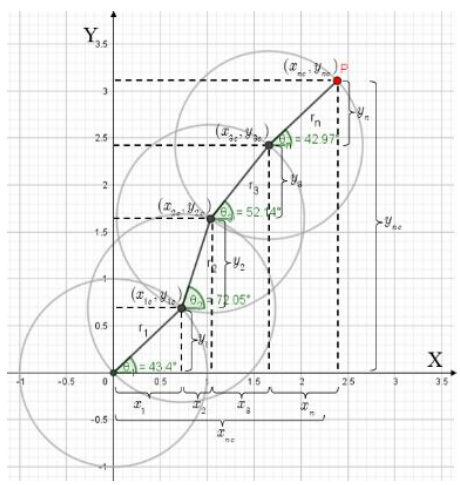


Figure 8 set of n circles

RESULTS

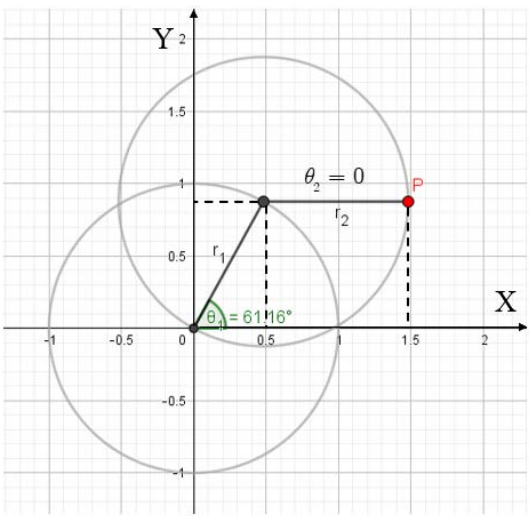
From the operating, it is possible that;

1. Total number of different locus

Out of a total of 1,000 locus, divided into 172 groups with different locus characteristics. Each group had the same locus characteristics, but different ration of angular velocity

2. Creating a Locus Equation

2.1 The locus equation of a set of 2 circles



Let r_1 be the length of $\overline{AB}$ is the radius of the first circle and r_2 be the length of $\overline{BP}$ is the radius of the second circle

Let ratio of angular velocity $\overline{AB}$ and $\overline{BP}$ are $\omega_1 : \omega_2$

Figure 9 set of 2 circles in case $\omega_2 = 0$

consider in case $\omega_2 = 0$

over t observe $\overline{AB}$ make an angle θ_1 and X-axis and $\overline{BP}$ make an angle 0 degree and X-axis

make it known that the angle of $\overline{BP}$ does not change according to the endpoint of $\overline{AB}$ is a pivot point

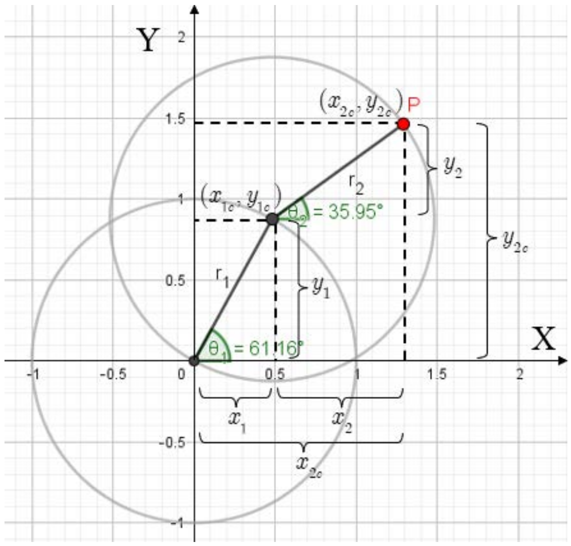


Figure 10 set of 2 circles in case $\omega_2 > 0$

consider in case $\omega_2 > 0$

overtime t get $\omega_1 t : \omega_2 t$

from $\theta_1 = \omega_1 t$ และ $\theta_2 = \omega_2 t$

get $\omega_1 t : \omega_2 t = \theta_1 : \theta_2$

$$\frac{\theta_1}{\theta_2} = \frac{\omega_1 t}{\omega_2 t}$$

thus $\theta_2 = \frac{\omega_2}{\omega_1}\theta_1$

from figure 10 from $x_2 = r_2 \cos\theta_2 = r_2 \cos\left(\frac{\omega_2}{\omega_1}\theta_1\right)$

$$y_2 = r_2 \sin\theta_2 = r_2 \sin\left(\frac{\omega_2}{\omega_1}\theta_1\right)$$

get $x_{2c} = x_1 + x_2$

$$y_{2c} = y_1 + y_2$$

get
$$\begin{aligned}(x_{2c}, y_{2c}) &= (r_1\cos\theta_1 + r_2\cos\theta_2, r_1\sin\theta_1 + r_2\sin\theta_2)\\ &= (r_1\cos(\omega_1 t) + r_2\cos(\omega_2 t), r_1\sin(\omega_1 t) + r_2\sin(\omega_2 t))\\ &= \left(r_1\cos\theta_1 + r_2\cos\left(\frac{\omega_2}{\omega_1}\theta_1\right), r_1\sin\theta_1 + r_2\sin\left(\frac{\omega_2}{\omega_1}\theta_1\right)\right)\end{aligned}$$

2.2 The locus equation of a set of 3 circles

To create an equation from a set of 3 circles, add $\overline{CP}$ and angle θ_3

Let ratio of angular velocity $\overline{AB}$, $\overline{BC}$ and $\overline{CP}$ are $\omega_1 : \omega_2 : \omega_3$ respectively

Let r_1 be the length of $\overline{AB}$ is the radius of the first circle , r_2 be the length of is the radius of the second circle และ r_3 be the length of $\overline{CP}$ is the radius of the third circle

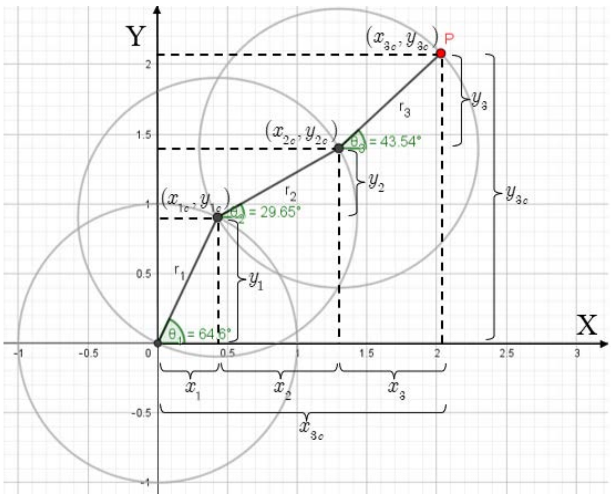


Figure 11 set of 3 circles

from $\theta_1 = \omega_1 t$, $\theta_2 = \omega_2 t$ และ $\theta_3 = \omega_3 t$

get $\omega_1 t : \omega_2 t : \omega_3 t = \theta_1 : \theta_2 : \theta_3$

that is $\dfrac{\theta_1}{\theta_2} = \dfrac{\omega_1 t}{\omega_2 t}$, $\dfrac{\theta_1}{\theta_3} = \dfrac{\omega_1 t}{\omega_3 t}$

$$\frac{\theta_1}{\theta_2} = \frac{\omega_1}{\omega_2}, \quad \frac{\theta_1}{\theta_3} = \frac{\omega_1}{\omega_3}$$

Thus $\theta_2 = \dfrac{\omega_2}{\omega_1}\theta_1$ and $\theta_3 = \dfrac{\omega_3}{\omega_1}\theta_1$

from figure 11 get
$$x_3 = r_3 \cos\theta_3 = r_3 \cos\left(\frac{\omega_3}{\omega_1}\theta_1\right)$$
$$y_3 = r_3 \sin\theta_3 = r_3 \sin\left(\frac{\omega_3}{\omega_1}\theta_1\right)$$

from
$$x_{3c} = x_1 + x_2 + x_3$$
$$y_{3c} = y_1 + y_2 + y_3$$

get

$$\begin{aligned}(x_{3c}, y_{3c}) &= (r_1 \cos\theta_1 + r_2 \cos\theta_2 + r_3 \cos\theta_3, r_1 \sin\theta_1 + r_2 \sin\theta_2 + r_3 \sin\theta_3) \\ &= (r_1 \cos(\omega_1 t) + r_2 \cos(\omega_2 t) + r_3 \cos(\omega_3 t), r_1 \sin(\omega_1 t) + r_2 \sin(\omega_2 t) + r_3 \sin(\omega_3 t)) \\ &= \left(r_1 \cos\theta_1 + r_2 \cos\left(\frac{\omega_2}{\omega_1}\theta_1\right) + r_3 \cos\left(\frac{\omega_3}{\omega_1}\theta_1\right), r_1 \sin\theta_1 + r_2 \sin\left(\frac{\omega_2}{\omega_1}\theta_1\right) + r_3 \sin\left(\frac{\omega_3}{\omega_1}\theta_1\right)\right)\end{aligned}$$

2.3 The locus equation of a set of n circles

From studying the equations of the set of 2 circles and set of 3 circles, the equation of the set of circles has been obtained.

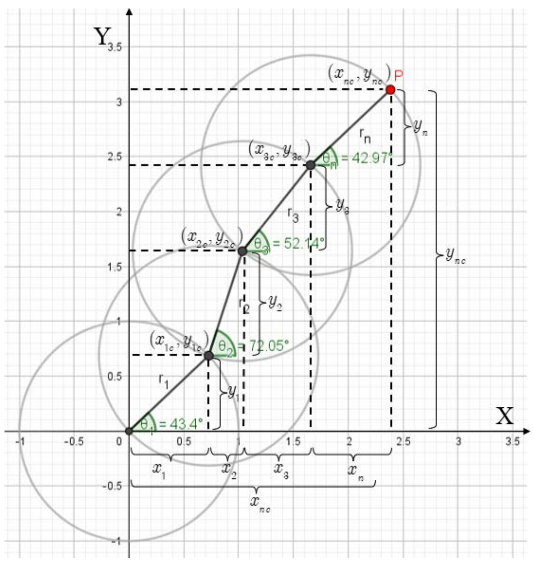


Figure 12 set of n circle

from $\theta_1 = \omega_1 t$, $\theta_2 = \omega_2 t$, ..., $\theta_n = \omega_n t$

get $\omega_1 t : \omega_2 t : ... : \omega_n t = \theta_1 : \theta_2 : ... : \theta_n$

$$\frac{\theta_1}{\theta_2} = \frac{\omega_1 t}{\omega_2 t} , \frac{\theta_1}{\theta_3} = \frac{\omega_1 t}{\omega_3 t} , \dots , \frac{\theta_1}{\theta_n} = \frac{\omega_1 t}{\omega_n t}$$

$$\frac{\theta_1}{\theta_2} = \frac{\omega_1}{\omega_2} , \frac{\theta_1}{\theta_3} = \frac{\omega_1}{\omega_3} , \dots , \frac{\theta_1}{\theta_n} = \frac{\omega_1}{\omega_n}$$

thus $\theta_2 = \frac{\omega_2}{\omega_1}\theta_1$, $\theta_3 = \frac{\omega_3}{\omega_1}\theta_1$, ..., $\theta_n = \frac{\omega_n}{\omega_1}\theta_1$

from $x_{nc} = x_1 + x_2 + x_3 + ... + x_n$

$$y_{nc} = y_1 + y_2 + y_3 + ... + y_n$$

get
$$x_{nc} = r_1 \cos\theta_1 + r_2 \cos\theta_2 + r_3 \cos\theta_3 + ... + r_n \cos\theta_n$$
$$y_{nc} = r_1 \sin\theta_1 + r_2 \sin\theta_2 + r_3 \sin\theta_3 + ... + r_n \sin\theta_n$$
$$x_{nc} = r_1 \cos\ \omega_1 t\ + r_2 \cos\ \omega_2 t\ + r_3 \cos\ \omega_3 t\ + ... + r_n \cos\ \omega_n t$$
$$y_{nc} = r_1 \sin\ \omega_1 t\ + r_2 \sin\ \omega_2 t\ + r_3 \sin\ \omega_3 t\ + ... + r_n \sin\ \omega_n t$$
$$x_{nc} = r_1 \cos\theta_1 + r_2 \cos\left(\frac{\omega_2}{\omega_1}\theta_1\right) + r_3 \cos\left(\frac{\omega_3}{\omega_1}\theta_1\right) + ... + r_n \cos\left(\frac{\omega_n}{\omega_1}\theta_1\right)$$
$$y_{nc} = r_1 \sin\theta_1 + r_2 \sin\left(\frac{\omega_2}{\omega_1}\theta_1\right) + r_3 \sin\left(\frac{\omega_3}{\omega_1}\theta_1\right) + ... + r_n \sin\left(\frac{\omega_n}{\omega_1}\theta_1\right)$$

Theorem 1 The locus equation of a set of n circles centered on X-axis or starting angle between the radius of i-th circle and X-axis with radius r_i and the ratio of angular velocity is $\omega_1 : \omega_2 : \omega_3 : ... : \omega_n$

$$(x_{nc}, y_{nc}) = \left(\sum_{i=1}^{n} r_i \cos\theta_i, \sum_{i=1}^{n} r_i \sin\theta_i\right) = \left(\sum_{i=1}^{n} r_i \cos(\omega_i t), \sum_{i=1}^{n} r_i \sin(\omega_i t)\right)$$
$$= \left(\sum_{i=1}^{n} r_i \cos\left(\frac{\omega_i}{\omega_1}\theta_1\right), \sum_{i=1}^{n} r_i \sin\left(\frac{\omega_i}{\omega_1}\theta_1\right)\right)$$

From theorem 1 Consider a counterclockwise rotation with an angle θ_i

And likewise, this equation will be true for a counterclockwise rotation with an angle $-\theta_i$

$$(x_{nc}, y_{nc}) = \left(\sum_{i=1}^{n} r_i \cos(\pm\theta_i), \sum_{i=1}^{n} r_i \sin(\pm\theta_i) \right) \text{ where } \theta_i = \omega_i t = \frac{\omega_i}{\omega_1}\theta_1$$

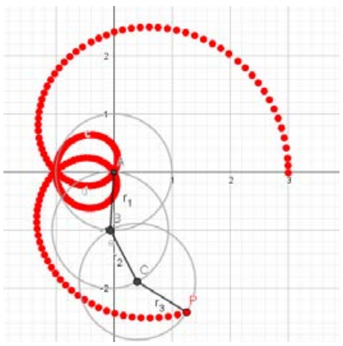


Figure 14 The locus formed by turning counter-clockwise

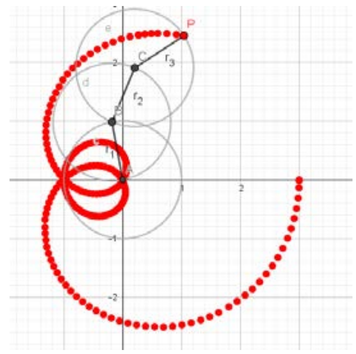
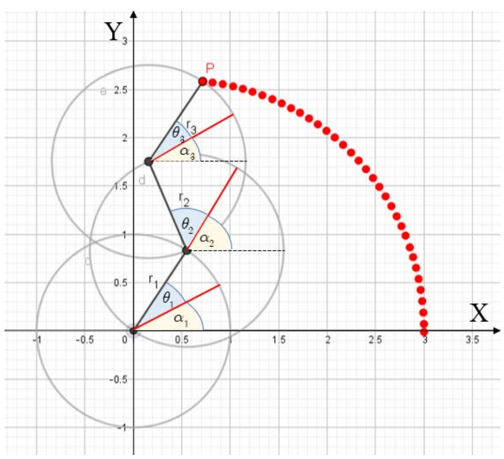


Figure 13 The locus formed by turning clockwise

Corollary 1 The locus equation of a set of n-th circles with an angle starting $\alpha_i \neq 0$

$$(x_{nc}, y_{nc}) = \left(\sum_{i=1}^{n} r_i \cos(\alpha_i \pm \theta_i), \sum_{i=1}^{n} r_i \sin(\alpha_i \pm \theta_i) \right) \text{ where } \theta_i = \omega_i t = \frac{\omega_i}{\omega_1}\theta_1$$



From figure 21 let $\gamma_i = \alpha_i \pm \theta_i$ when i=1, 2, 3, …, n

From theorem 1, the locus equation is

$$(x_{nc}, y_{nc}) = \left(\sum_{i=1}^{n} r_i \cos(\gamma_i), \sum_{i=1}^{n} r_i \sin(\gamma_i) \right)$$

$$= \left(\sum_{i=1}^{n} r_i \cos(\alpha_i \pm \theta_i), \sum_{i=1}^{n} r_i \sin(\alpha_i \pm \theta_i) \right)$$

Figure 15 Locus from set of n circles when $\alpha_i \neq 0$

Corollary 2 From theorem 1, when the ratio of angular velocity is equal, the locus will be circular

proof let the angular velocity each point is ω

that is $\omega_1 = \omega_2 = \omega_3 = \ldots = \omega_n = \omega$ then

$$(x_{nc}, y_{nc}) = \left(\sum_{i=1}^{n} r_i \cos(\omega_i t), \sum_{i=1}^{n} r_i \sin(\omega_i t)\right) = \left(\sum_{i=1}^{n} r_i \cos(\omega t), \sum_{i=1}^{n} r_i \sin(\omega t)\right)$$

$$= \left(\left(\sum_{i=1}^{n} r_i\right)\cos(\omega t), \left(\sum_{i=1}^{n} r_i\right)\sin(\omega t)\right) = (R\cos(\omega t), R\sin(\omega t)) \qquad \text{where } R = \sum_{i=1}^{n} r_i$$

Therefore, the resulting locus is a circular equation with radius R

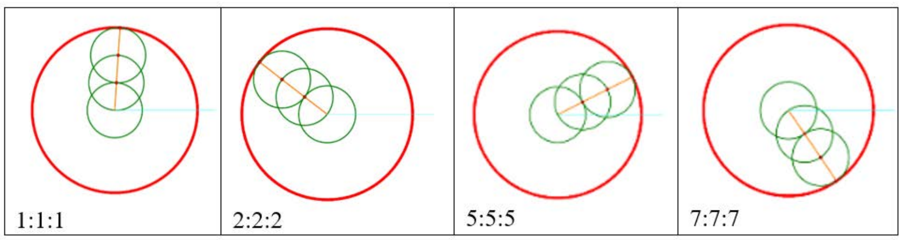


Figure 16 Locus with the ratio of angular velocity is equal

Corollary 3 from theorem 1, when ratio of angular velocity is a minimum ratio when multiplied by a constant, and get the same pattern

proof Let the ratio of angular velocity is

$\varepsilon_1 : \varepsilon_2 : \ldots : \varepsilon_n = k(\omega_1 : \omega_2 : \ldots : \omega_n) = k\omega_1 : k\omega_2 : \ldots : k\omega_n$ then

$$(x_{nc}, y_{nc}) = \left(\sum_{i=1}^{n} r_i \cos\left(\frac{\varepsilon_i}{\varepsilon_1}\theta_1\right), \sum_{i=1}^{n} r_i \sin\left(\frac{\varepsilon_i}{\varepsilon_1}\theta_1\right)\right) = \left(\sum_{i=1}^{n} r_i \cos\left(\frac{k\omega_i}{k\omega_1}\theta_1\right), \sum_{i=1}^{n} r_i \sin\left(\frac{k\omega_i}{k\omega_1}\theta_1\right)\right)$$

$$= \left(\sum_{i=1}^{n} r_i \cos\left(\frac{\omega_i}{\omega_1}\theta_1\right), \sum_{i=1}^{n} r_i \sin\left(\frac{\omega_i}{\omega_1}\theta_1\right)\right)$$

Therefore, the locus has the same pattern with the locus of the angular ratio $\omega_1 : \omega_2 : \ldots : \omega_n$

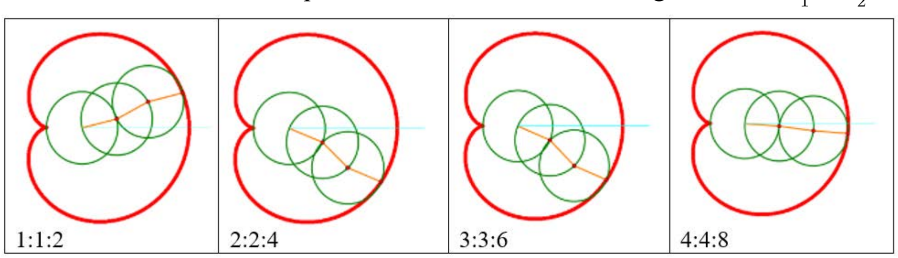


Figure 17 locus with a number of angular velocity ratios k times the original angular velocity ratio

Corollary 4 from theorem 1 when the radius was the same, take the ratios permutation and got the same locus pattern

proof Let the radius of the circle is $r_i = r$ i=1, 2, 3, …, n and let $\omega_{P_1} : \omega_{P_2} : \ldots : \omega_{P_n}$ is a ratio of angular velocity resulting from permutation $\omega_1 : \omega_2 : \ldots : \omega_n$ then

$$\begin{aligned}(x_{nc}, y_{nc}) &= \left(\sum_{i=1}^{n} r_i \cos(\omega_{P_i} t), \sum_{i=1}^{n} r_i \sin(\omega_{P_i} t)\right) = \left(\sum_{i=1}^{n} r \cos(\omega_{P_i} t), \sum_{i=1}^{n} r \sin(\omega_{P_i} t)\right) \\ &= \left(r\sum_{i=1}^{n} \cos(\omega_{P_i} t), r\sum_{i=1}^{n} \sin(\omega_{P_i} t)\right) = \left(r\sum_{i=1}^{n} \cos(\omega_i t), r\sum_{i=1}^{n} \sin(\omega_i t)\right) \\ &= \left(\sum_{i=1}^{n} r \cos(\omega_i t), \sum_{i=1}^{n} r \sin(\omega_i t)\right)\end{aligned}$$

Therefore, the locus has the same pattern

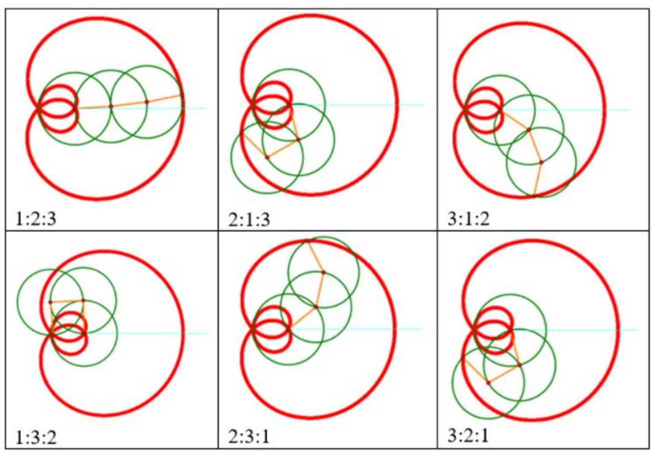


Figure 18 locus with the same radius of all circles and permutation of angular velocity

4.3 check the equation

When creating a locus from the above equation in GeoGebra and create a trace of movement from a set of circles by having the same locus characteristics as the above equation then the traces of motion overlap with the locus obtained from the equation.

Interpretation of results

From the locus equation, it was found that

1. Locus formed by $P(x,y)$ moving on the circumference in set of circles the same locus due to the use of commutative property for addition and multiplying by a constant in the equation

2. Symmetrical pattern locus due to rotation of giving the center of all circles and points in the same straight line therefore, the resulting locus will look symmetrical. An axis of symmetry is a straight line that passes through the center of every circle and P point. Because when θ substituted by $-\theta$ in the above equation, then the equation that is equivalent to the original equation

3. Locus with the same ratio of angular velocity will be circular because the angular velocity of each point is equal, all points move in the same straight line. Therefore, it forms a circle.

Conclusions

After finding locus equation of the point on circumference of the n -th circle that formed by moving the center of any radius circles on the outermost circumference of preceding set of circles, the general equation in the set of circles can be showed as

Theorem The locus equation of a set of n -th circles.

$$\begin{aligned}(x_{nc}, y_{nc}) &= \left(\sum_{i=1}^{n} r_i \cos(\alpha_i \pm \theta_i), \sum_{i=1}^{n} r_i \sin(\alpha_i \pm \theta_i)\right) \\ &= \left(\sum_{i=1}^{n} r_i \cos(\alpha_i \pm \omega_i t), \sum_{i=1}^{n} r_i \sin(\alpha_i \pm \omega_i t)\right) \\ &= \left(\sum_{i=1}^{n} r_i \cos\left(\alpha_i \pm \frac{\omega_i}{\omega_1}\theta_1\right), \sum_{i=1}^{n} r_i \sin\left(\alpha_i \pm \frac{\omega_i}{\omega_1}\theta_1\right)\right)\end{aligned}$$

Where r_i is the radius of i -th circle, θ_i is an angle between the radius of i -th circle and X-axis, ω_i is the angular velocity, t is elapsed time and α_i is a starting angle between the radius of i -th circle and X-axis.

Feature development

1. we can predict the position of moving objects, such as planets, satellites, moon of the planets in the universe. In astronomy, it can be seen that it will look like 3d if we can make circular motions in 2d by using projection or other methods. This makes it similar to set of circles, allowing to apply our equations by substituting variables such as distance between planets, starting angle and angular velocity in order to predict the location of the desired object

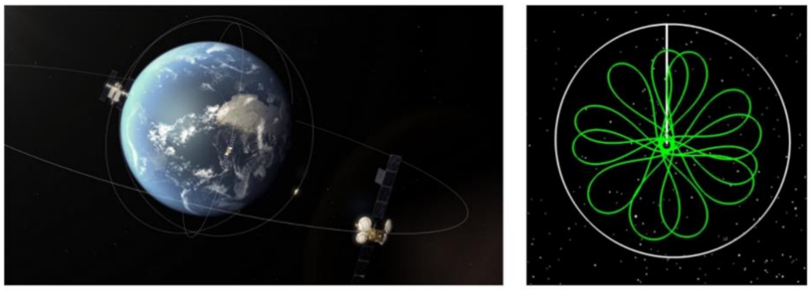


Figure 19 Predict the position of IBEX satellite orbit.

2. As observed Dream catchers are ancient Indian handicrafts that are believed to filter dreams so that good dreams stay with you and nightmares disappear. Which looks like a circle Woven with ropes in various patterns and geometric designs in Islamic art are often made up of repeated combinations of squares and circles, which may overlap and intertwine. as well as Arab to create intricate and intricate patterns, as well as a wide variety of tessellations, these patterns resemble our locus by using human labor and local wisdom to create them. So, I thought of creating a pattern designer that would allow for more patterns and more convenience.   It also maintains a culture for the new generation to be more easily accessible.

Figure 20 Indian dream catcher

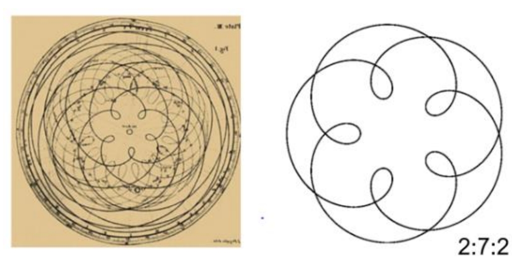


Figure 21 Art of Islamic pattern

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In this project, there are n sticks connected end-to-end to form a piecewise linear curve. Each stick rotates with a designated angular velocity with respect to the end connecting the the preceding stick. The locus equation of the point on circumference of the final n-th circle is studied. A preliminary numerical experiment was first conducted using computer graphic software GSP, where 1000 samples were run to see the patterns. Then, mathematical derivations about the locus of the n^th stick using linear property were done. Basically, this is an interesting problem with many applications, and should be encouraged as it is not easy for high school students.